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DARK ENERGY EQUATION OF STATE AND COSMIC TOPOLOGY

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The immediate observational consequence of a non-trivial spatial topology of the Universe is that an observer could potentially detect multiple images of radiating sources. In particular, a non-trivial topology will generate pairs of correlated circles of temperature fluctuations in the anisotropies maps of the cosmic microwave background (CMB), the so-called circles-in-the-sky. In this way, a detectable non-trivial spatial topology may be seen as an observable attribute, which can be probed through the circles-in-the-sky for all locally homogeneous and isotropic universes with no assumptions on the cosmological dark energy (DE) equation of state (EOS) parameters. We show that the knowledge of the spatial topology through the circles-in-the-sky offers an effective way of reducing the degeneracies in the DE EOS parameters. We concretely illustrate the topological role by assuming a Poincaré dodecahedral space topology and reanalyzing the constraints on the parameters of a specific EOS which arise from the supernovae type Ia, baryon acoustic oscillations and the CMB plus the statistical topological contribution.

Keywords: Dark energy equation of state parameters; cosmic topology.

1. Introduction

In the standard cosmology, the Universe is modelled by a space-time manifold $\mathcal{M}_4 = \mathbb{R} \times M_3$ endowed with a locally (spatially) homogeneous and isotropic Robertson-Walker metric

$$ds^2 = -c^2 dt^2 + a^2(t) [d\chi^2 + S_k^2(\chi)(d\theta^2 + \sin^2 \theta d\phi^2)], \quad (1)$$

where $a(t)$ is the cosmological scale factor, and $S_k(\chi) = (\chi, \sin \chi, \sinh \chi)$ depending on the sign of the constant curvature ($k = 0, 1, -1$) of the spatial sections M_3 . The 3-space M_3 is usually taken to be one of the simply-connected manifolds: Euclidean \mathbb{R}^3 , spherical \mathbb{S}^3 , or hyperbolic \mathbb{H}^3 . However, it is known that the great majority of such constant curvature 3-spaces are multiply-connected quotient manifolds of the form \mathbb{R}^3/Γ , \mathbb{S}^3/Γ , and \mathbb{H}^3/Γ , where Γ is a fixed-point free group of isometries of the corresponding covering space. On the other hand, given that the connectedness of the spatial sections M_3 has not been determined by cosmological observations, and

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since geometry does not fix the topology, our 3-dimensional space may be one of these possible multiply-connected quotient manifolds (for reviews on this issue see, e.g., the Refs. 1).

The immediate observational consequence of a detectable nontrivial topology² of M_3 is the existence of the circles-in-the-sky, i.e., pairs of matching circles will be imprinted on the cosmic microwave background (CMB) anisotropy sky maps.³ Hence, to observationally probe a putative nontrivial topology of M_3 , one should examine the CMB maps in order to extract the pairs of correlated circles and determine the spatial topology.

In this regard, in a few recent works⁴ in the context of Λ cold dark matter (Λ CDM) model, it has been shown that the knowledge of the spatial topology through the circles-in-the-sky offers an effective way of setting constraints on the density parameters associated with matter (Ω_m) and dark energy (Ω_Λ). In other words, it has been shown in Refs. 4 that specific circles-in-the-sky detectable topology of the spatial section of the Universe can be used to reduce the degeneracies in the density parameters plane $\Omega_m - \Omega_\Lambda$, which arise from statistical analyses with data from current observations (see also the related Refs. 5).

The question as to whether the detection of a non-trivial cosmic topology can also be used to set constraints on the equation of state parameters naturally arises here. In this paper we address this question by examining to what extent a non-trivial spatial topology can be used to place further constraints on the dark energy (DE) equation of state parameter w_x . To this end, we shall assume the Poincaré dodecahedral space as the circles-in-the-sky observable spatial topology, and reanalyze the current constraints on a two-parameter DE equation of state (EOS) of Ref. 6, which arise from the Type Ia supernovae (SNe Ia) data from the Legacy sample⁷ along with the baryon acoustic oscillations (BAO) peak in the large-scale correlation function of the Sloan Digital Sky Survey (SDSS),⁸ and CMB shift parameter.⁹

2. Poincaré Dodecahedral Space

Combining the first-year CMB with other astronomical data, the WMAP team¹⁰ reported the estimated value of the total density $\Omega_{\text{tot}} = 1.02 \pm 0.02$ (1σ level), which includes a positively-curved universe as a possibility. This reported sign for the curvature density ($\Omega_k = 1 - \Omega_{\text{tot}} \leq 0$) has been reinforced by the combination of WMAP three year CMB measurements with other observational data sets, cf. Table 12 of Ref. 11, wherein six different values for the Ω_{tot} ranging from a very nearly flat ($\Omega_{\text{tot}} = 1.005^{+0.0060}_{-0.0068}$) to positively curved ($\Omega_{\text{tot}} = 1.023 \pm 0.014$) depending on the combination of data sets used to resolve the geometrical degeneracy.

In this work we assume the positively-curved Poincaré dodecahedral space (PDS) as the circles-in-the-sky detectable topology of the spatial sections of the Universe.^a

^aThis topology is consistent with the observed total density, and accounts for both the suppression of power of the low multipoles of the first and three year WMAP data^{10,11} and the temperature two-point correlation function (see Refs. 12–14).

The fundamental polyhedron (FP) of the PDS topology is a regular spherical dodecahedron that tiles the covering space \mathbb{S}^3 into 120 identical cells. The radius r_{inj} (called the injectivity radius) of the smallest sphere ‘inscribable’ in the FP is $\pi/10$. The PDS is globally homogeneous, and a detectable PDS topology ($\chi_{\text{lss}} > r_{\text{inj}}$, cf. Ref. 2) gives rise to six pairs of antipodal circles-in-the-sky, one of which is shown in Fig 1.

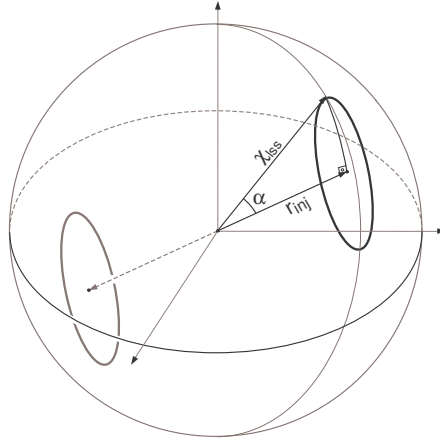


Fig. 1. A schematic illustration of a pair of antipodal matching circles on the sphere of last scattering. The relation between the angular radius α of the circles and the angular sides r_{inj} and χ_{lss} is given by Eq.(2).

Now, a straightforward use of a Napier’s rule to the right-angled spherical triangle furnishes a relation between the angular radius α and the angular sides r_{inj} and χ_{lss} of the last scattering sphere, namely

$$\cos \alpha = \frac{\tan r_{\text{inj}}}{\tan \chi_{\text{lss}}}, \quad (2)$$

which can be solved for χ_{lss} to give

$$\chi_{\text{lss}} = \tan^{-1} \left[\frac{\tan r_{\text{inj}}}{\cos \alpha} \right]. \quad (3)$$

This equation shows that for a given topology, the measurement of a value α for a circle-in-the-sky radius gives the distance χ_{lss} to the last scattering surface in *units of the curvature radius* today, $a_0 = a(t_0) = (H_0 \sqrt{|1 - \Omega_{\text{tot}}|})^{-1}$. Given a detection of a circle-in-the-sky, one can further constrain the EOS parameters, as we shall show in Sec. 4, by comparing χ_{lss} given by Eq. (3) with that predicted by a model with specific EOS parametrization.

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3. Dark Energy Parametrization

In the light of current observations, we assume the Friedmann-Lemaître-Robertson-Walker (FLRW) framework in which the current matter content of the Universe is well approximated by a dust of density ρ_m (baryonic plus dark matter) along with a dark energy perfect fluid component of density ρ_x and pressure p_x . The Friedmann equation is then given by

$$H^2 = \frac{8\pi G}{3c^2}(\rho_m + \rho_x) - \frac{kc^2}{a^2}, \quad (4)$$

where $H = \dot{a}/a$ is the Hubble parameter, overdot stands for derivative with respect to time t , G is Newton's constant, and c is the speed of light.

If one further assumes that these fluid components do not interact, the energy conservation equations reduce to

$$\dot{\rho}_i + 3H(\rho_i + p_i) = 0, \quad (5)$$

for each component $i = m, x$. This equation can be rewritten in terms of the redshift $1 + z = a_0/a$ and the equation of state parameters $w_i = p_i/\rho_i$ in the form

$$\frac{d\rho_i}{dz} = \frac{3\rho_i(1 + w_i)}{1 + z}, \quad (6)$$

and has an integral solution

$$\rho_i(z) = \rho_i(0) \exp \left[\int_0^z \frac{3(1 + w_i)}{1 + z'} dz' \right]. \quad (7)$$

In this way, the Friedmann equation (4) can now be rewritten in terms of the redshift z and the dimensionless density variables $\Omega_i = \rho_i(0)/\rho_{crit}(0)$, where $\rho_{crit}(z) = 3c^2 H^2(z)/(8\pi G)$, in the form

$$E^2(z) = \Omega_m(1 + z)^3 + \Omega_k(1 + z)^2 + \Omega_x f(z), \quad (8)$$

where

$$f(z) = \exp \left[\int_0^z \frac{3(1 + w_x(z'))}{1 + z'} dz' \right], \quad (9)$$

and where $E(z) = H(z)/H_0$ is the dimensionless Hubble function and $\Omega_k = -kc^2/(a_0^2 H_0^2)$ is the density curvature parameter.

In this work we shall consider a model that is defined by Eq. (8) and Eq. (9) along with the Jassal-Bagla-Padmanabhan⁶ (JBP) dark energy parametrization

$$w_x(z) = w_0 + \frac{w_1 z}{(1 + z)^2}, \quad (10)$$

which combined with Eqs. (8) and (9) give

$$E^2(z) = \Omega_m(1 + z)^3 + \Omega_k(1 + z)^2 + \Omega_x(1 + z)^{3(1+w_0)} \exp \left[\frac{3w_1 z^2}{2(1 + z)^2} \right]. \quad (11)$$

We note that for $z = 0$ one has the constraint equation $\Omega_x = 1 - \Omega_k - \Omega_m$.

From Eq. (3) it is clear that to study the constraints on the parameters of the JPB model we need the theoretical value of χ_{ls} predicted by this model, which is given by

$$\chi_{\text{ls}} = \sqrt{|\Omega_k|} \frac{d_c(z_{\text{ls}})}{d_H} = \sqrt{|\Omega_k|} \int_0^{z_{\text{ls}}} \frac{dz'}{E(z')}, \quad (12)$$

where $d_c(z) = d_H \int_0^z dz'/E(z')$ is the comoving distance, $d_H = c/H_0$ is the Hubble distance today, and z_{ls} is the redshift of the last scattering surface.

4. Observational Constraints

To illustrate the role played by the PDS topology in constraining the EOS parameters of the model discussed in the previous section, we have reanalyzed, with and without the topological contribution, the constraints on the parameters that arise from Legacy sample⁷ of SNe Ia along with the baryon acoustic oscillations (BAO) peak position in the galaxy power spectrum,⁸ and CMB shift parameter.⁹

For the sake of brevity, we shall not discuss here the likelihoods for the SNe Ia, BAO and CMB data, and simply refer to them as $L_{\text{SN}}(\vec{P})$, $L_{\text{BAO}}(\vec{P})$ and $L_{\text{CMB}}(\vec{P})$ with $\vec{P} \equiv (\Omega_m, \Omega_k, w_0, w_1)$. Further details on these likelihoods can be found in a number of papers as, for example, in Refs. 4b and 5b. In this way, we focus on the likelihood of topological origin, which is included in the statistical analysis as a gaussian prior as follows. The measurement of a circle-in-the-sky of radius α in CMB map involves unavoidably observational uncertainty σ_α . These observational pieces of topological information can be included in the statistical analyses by using that $r_{\text{inj}} = \pi/10$ (for PDS topology) along with Eq. (3) to have χ_{ls} and $\sigma_{\chi_{\text{ls}}}$ (through the usual error propagation formula). In this way, a detection of a PDS circles-in-the-sky would provide an observational value of the radius of the last scattering surface and the associated uncertainty, i.e. $\chi_{\text{ls}} \pm \sigma_{\chi_{\text{ls}}}$, which we compare with the theoretical values predicted by the JPB model by assuming a gaussian distribution with the mean value given by Eqs. (11) and (12). Thus, the likelihood associated to the topology is given by

$$L_{\text{Top}}(\vec{P}) \propto \exp \left\{ -\frac{1}{2} \frac{\left[\chi_{\text{ls}} - \sqrt{|\Omega_k|} \frac{d_c(z_{\text{ls}})}{d_H} \right]^2}{(\sigma_{\chi_{\text{ls}}})^2} \right\}, \quad (13)$$

whose dependence on the model parameters $\vec{P} = (\Omega_m, \Omega_k, w_0, w_1)$ clearly comes explicitly from Ω_k and from the comoving distance $d_c(z_{\text{ls}})$.

5. Results and Conclusions

In order to make apparent the role of the topological term L_{Top} in constraining the equation of state parameters, we have performed the joint analysis maximizing the

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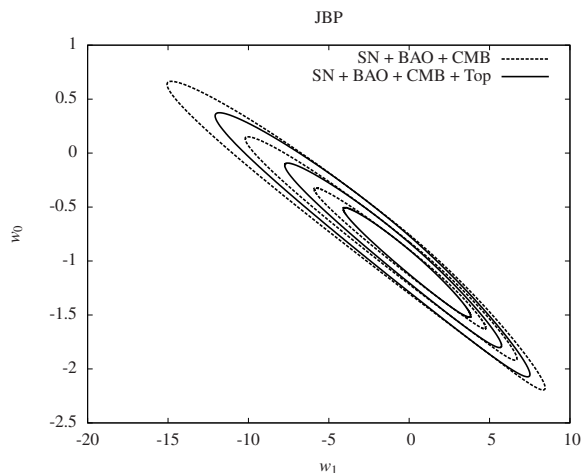


Fig. 2. Confidence regions for 1σ – 3σ in the w_1 – w_0 plane for JBP parametrization. The dashed and full lines indicate the confidence regions without and with the topological term $L_{\text{Top}}(\vec{P})$. Clearly the PDS topology reduces the degeneracies in the EOS parameters w_0 and w_1 which arise from the SNe Ia, BAO and CMB.

likelihoods with and without the topological term, namely,

$$L_1(\vec{P}) = L_{\text{SN}}(\vec{P}) L_{\text{BAO}}(\vec{P}) L_{\text{CMB}}(\vec{P}) L_{\text{Top}}(\vec{P}), \quad (14)$$

$$L_2(\vec{P}) = L_{\text{SN}}(\vec{P}) L_{\text{BAO}}(\vec{P}) L_{\text{CMB}}(\vec{P}), \quad (15)$$

where $L_{\text{Top}}(\vec{P})$ is given by a circle-in-the-sky radius along with an unavoidable observational uncertainty in its measurements, which we take to $\alpha = 20^\circ \pm 4^\circ$. This value for the radius was not covered by the search for almost antipodal correlated circles (with radii larger than $\alpha = 25^\circ$) made in Ref. 16, in which no circles were found.^b We also used a conservative error of 20% to account for the possible uncertainties in the measurement of the circle radius.

Figure 2 shows 1σ – 3σ confidence regions of the JBP parameter w_1 – w_0 plane calculated by using the above likelihoods $L_1(\vec{P})$ (with topology) and $L_2(\vec{P})$ (with no topology) and marginalizing over the remaining parameters Ω_m and Ω_k . It is clear from this figure that a circles-in-the-sky detection of the PDS topology reduces appreciably the parametric regions allowed by Legacy SNe Ia sample combined with the BAO peak position in the galaxy power spectrum, and CMB shift parameter. In Tab. 1 we explicitly quantify these constraints by collecting together the 3σ lower

^bWe note that another statistical technique for probing below the limit of 20° for a directed search for a *specific* topology was introduced in Ref. 17. By using this combined search procedure with the knowledge of four or more circle pairs, for example, one might be able to detect circles with radii as small as 5° .

and upper bounds of the parameters w_0 and w_1 which were calculated without and with the topological statistical term. Table 1 makes apparent that the allowed intervals for w_0 and w_1 are reduced, respectively, of $\sim 14.4\%$ and $\sim 16.7\%$ by the topological term.

Table 1. The lower and upper 3σ limits of in the $w_0 - w_1$ plane without and with L_{Top} .

Model	w_0	w_1
JBP	(-2.194, 0.665)	(-15.059, 8.457)
JBP+PDS	(-2.073, 0.374)	(-12.069, 7.510)

To complete this study we show in Figure 3 the results of $1\sigma - 3\sigma$ statistical analysis for $\Omega_k - w_0$ and $\Omega_k - w_1$ planes (again marginalizing over the other parameters). This figure makes apparent that the main role of the topology in these parameter planes is to reduce considerably the degeneracy on Ω_k allowed by the above-mentioned observational data, in agreement with earlier related results obtained in different contexts.⁵ We also note that the w_0 and w_1 3σ limits are the same on both Fig. 2 and Fig. 3. Finally, we note that it can be shown that a suitable Gaussian prior on the curvature density parameter Ω_k can mimic the role of the topology in such statistical analyses.

To conclude, we note that although we have illustrated the degree to which a circle-in-the-sky topology detection reduces the degeneracies in the DE EOS parameters by using $\alpha = 20^\circ \pm 4^\circ$, our general results should not change appreciably for smaller radius and uncertainties, and its general features should hold for other two-parameters EOS.

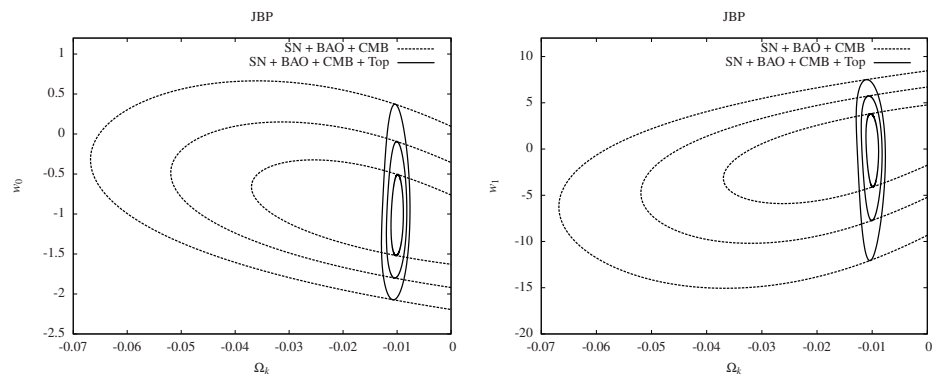


Fig. 3. Confidence regions in $1\sigma - 3\sigma$ of (Ω_k, w_0) and (Ω_k, w_1) for JBP parametrization. The main effect of the topological prior is to constrain the value of Ω_k .

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